Mechanizing Synthetic Tait Computability in Istari

MURI Meeting
August 5, 2025

Runming Li

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Mechanizing *Synthetic Tait Computability* in *Istari*

A categorical gluing approach to metatheory of programming languages using a modal dependent type theory

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A proof assistant based on Computational Type Theory, in the tradition of NuPRL

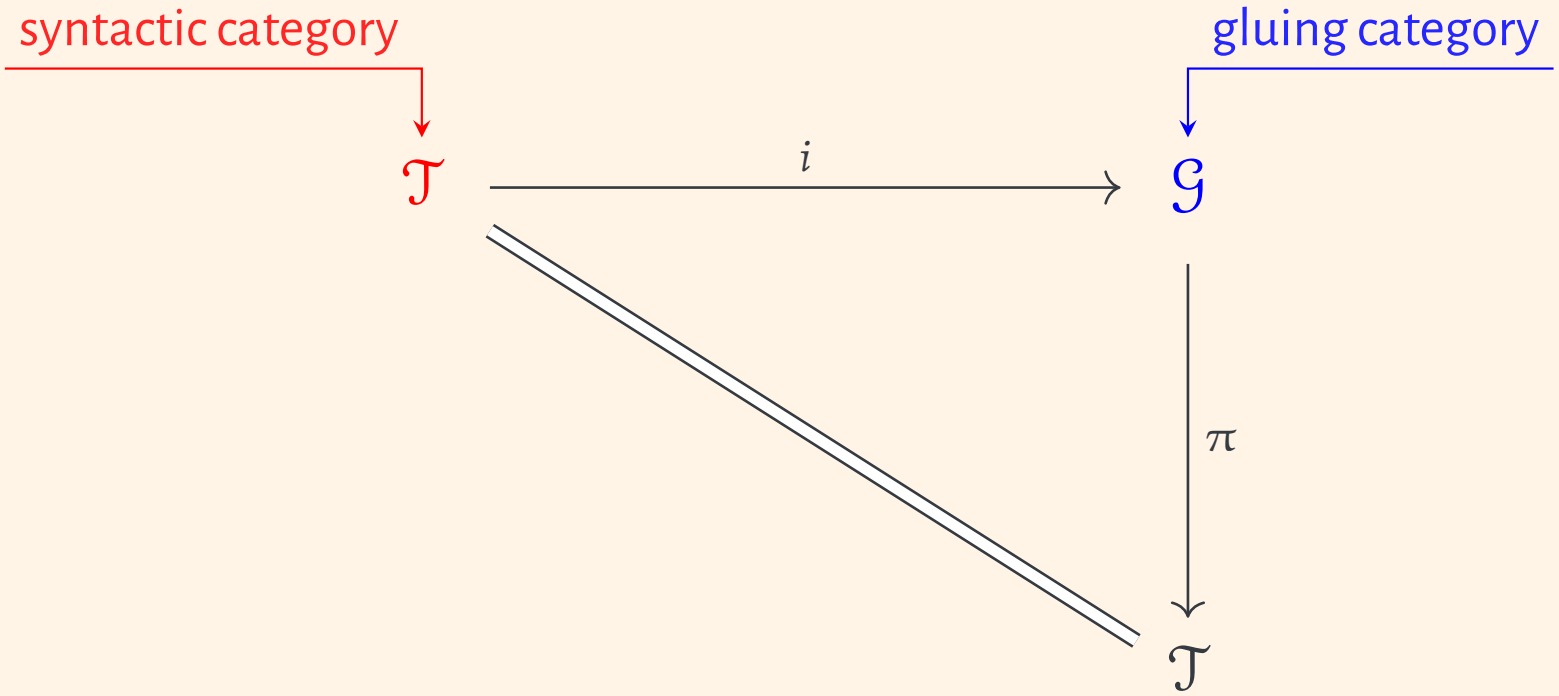
Canonicity via gluing

Start with a syntactic category $\mathcal{T}$.

Theorem (Canonicity)

For every $b : 1_{\mathcal{T}} \to tm_{\mathcal{T}}(bool_{\mathcal{T}})$, it must be the case that either $b = true_{\mathcal{T}}$ or $b = false_{\mathcal{T}}$.

A gluing category $\mathcal{G}$



Proof strategy:

- Construct a category $\mathcal{G}$;
- Find structure-preserving functors $i : \mathcal{T} \to \mathcal{G}$ and $\pi : \mathcal{G} \to \mathcal{T}$ such that $\pi \circ i = \mathrm{Id}_{\mathcal{T}}$.

A gluing category $\mathcal{G}$

$\Gamma = \mathsf{Hom}_{\mathcal{T}}(1_{\mathcal{T}}, -)$
the global section functor

$$\mathcal{G} = \mathbf{Set} \downarrow \Gamma$$

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Objects:

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Think as: proof-relevant predicates on closed terms of type A!

Canonicity proof

Define

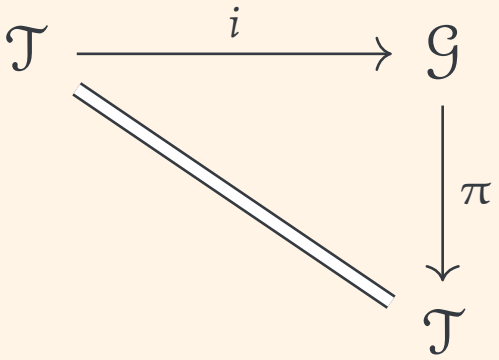
$$i : \mathcal{T} \to \mathcal{G}$$

$$i\,(\mathsf{tm}_{\mathcal{T}}(\mathsf{bool}_{\mathcal{T}})) = \mathsf{tm}_{\mathcal{G}}(\mathsf{bool}_{\mathcal{G}})$$

$$\vdots$$

$$\pi : \mathcal{G} \to \mathcal{T}$$

the forgetful functor



$$\pi \circ i = \mathsf{Id}_{\mathcal{T}}$$

Canonicity proof

Theorem (Canonicity)

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Proof

By examining the morphism $i(b) : 1_{\mathcal{G}} \to \mathsf{tm}_{\mathcal{G}}(\mathsf{bool}_{\mathcal{G}})$.

□

Close $\mathcal{G}$ under connectives

- Need to close $\mathcal{G}$ under dependent products, dependent sums, equality types, and so on.
- A lot of subtle details to check such as naturality.
- No mechanization until very recently by Kaposi and Pujet [4].

Sterling's Synthetic Tait Computability

Sterling [8] identifies that the internal language of the gluing category $\mathcal{G}$ is a modal extensional dependent type theory based on a synthetic phase distinction between syntax and semantics.

c.f. Harrison's talk this morning!

Sterling's Synthetic Tait Computability

Gluing proofs become programming and type-checking obligations in this language.

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Synthetic Tait Computability

Parametricity for an ML module calculus	Sterling & Harper [10]
Normalization for Cartesian Cubical Type Theory	Sterling & Angiuli [9]
Normalization for a multimodal type theory	Gratzer [3]
⋮	
Canonicity for Cost-Aware Logical Framework [6]	Li & Harper [5]

Synthetic Tait Computability

Parametricity for an ML module calculus — Sterling & Harper [10]

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⋮

If $e : \mathsf{F}(\mathsf{nat})$, then $e = \mathsf{charge}^c(\mathsf{ret}(\mathsf{suc}^n(\mathsf{zero})))$.

Canonicity for Cost-Aware Logical Framework [6] — Li & Harper [5]

A modal dependent type theory with a phase distinction between cost and behavior based on call-by-push-value.

Phase distinction between syntax and semantics

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$$\bullet A \triangleq A \sqcup \mathsf{syn}$$

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$$\begin{array}{ccc} A \times \mathsf{syn} & \xrightarrow{\pi_2} & \mathsf{syn} \\ {\scriptstyle \pi_1}\downarrow & \ulcorner & \downarrow{\scriptstyle \star} \\ A & \xrightarrow{\eta} & \bullet A \end{array}$$

$$\begin{aligned} &\mathsf{data}\ \bullet\ (A : \mathcal{U}) : \mathcal{U}\ \mathsf{where} \\ &\quad \eta : A \to \bullet A \\ &\quad \star : (z : \mathsf{syn}) \to \bullet A \\ &\quad \_ : (a : A)\ (z : \mathsf{syn}) \to \eta\, a = \star\, z \end{aligned}$$

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$\bigcirc\bullet A$ is contractible

Syntax

record $\mathbb{SIG}$ where

tp : $\mathcal{U}$

tm : tp $\to$ $\mathcal{U}$

bool : tp

true : tm bool

false : tm bool

Syntax

record $\mathbb{SIG}$ where

$\mathsf{tp} : \mathcal{U}$

$\mathsf{tm} : \mathsf{tp} \to \mathcal{U}$

$\mathsf{bool} : \mathsf{tp}$

$\mathsf{true} : \mathsf{tm}\ \mathsf{bool}$

$\mathsf{false} : \mathsf{tm}\ \mathsf{bool}$

$\mathsf{pi} : (A : \mathsf{tp}) \to (\mathsf{tm}\ A \to \mathsf{tp}) \to \mathsf{tp}$

$\mathsf{lam} : ((a : \mathsf{tm}\ A) \to \mathsf{tm}(B\ a)) \to \mathsf{tm}\ (\mathsf{pi}\ A\ B)$

$\mathsf{app} : \mathsf{tm}\ (\mathsf{pi}\ A\ B) \to (a : \mathsf{tm}\ A) \to \mathsf{tm}\ (B\ a)$

$\mathsf{pi}_\beta : \mathsf{app}\ (\mathsf{lam}\ f)\ x = f\ x$

$\mathsf{pi}_\eta : \mathsf{lam}\ (\mathsf{app}\ f) = f$

Semantics

Assume a model $\mathsf{M} : \mathbb{SIG}$ that represents the syntax.

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Construct a model $\mathsf{M} : \{\mathbb{SIG} \mid \mathsf{syn} \hookrightarrow \mathsf{M}\}$ that represents the semantics.

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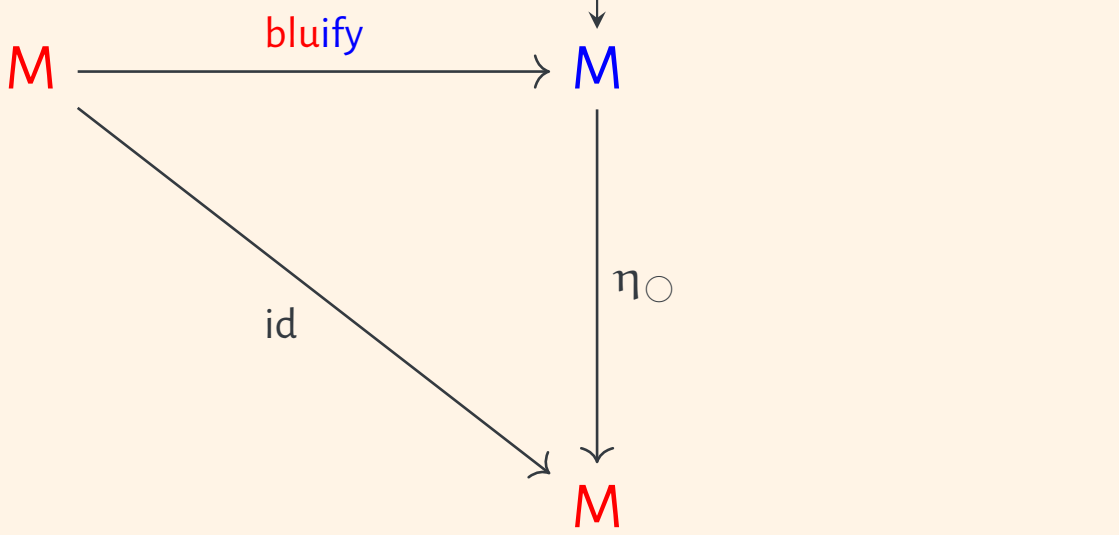
defined in the internal language of $\mathcal{G}$



Constructing M

$$\mathsf{tp} : \{\mathcal{U} \mid \mathsf{syn} \hookrightarrow \mathsf{tp}\}$$

$$\mathsf{tp} = \Sigma_{A:\mathsf{tp}} \{\mathcal{U} \mid \mathsf{syn} \hookrightarrow \mathsf{tm}\, A\}$$

$$\mathsf{tm} : \{\mathsf{tp} \to \mathcal{U} \mid \mathsf{syn} \hookrightarrow \mathsf{tm}\}$$

$$\mathsf{tm}\, A = \pi_2 A$$

Semantics of booleans is that it is either true or false.

$$\mathsf{bool} : \{\mathsf{tp} \mid \mathsf{syn} \hookrightarrow \mathsf{bool}\}$$

$$\mathsf{bool} = (\mathsf{bool}, \Sigma_{b:\mathsf{tm}(\mathsf{bool})} \bigcirc((b = \mathsf{true}) + (b = \mathsf{false})))$$

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$$\mathsf{pi}\, A\, B = (\mathsf{pi}\, A\, B, \Sigma_{e:\mathsf{tm}(\mathsf{pi}\, A\, B)} \{(a : \mathsf{tm}\, A) \to \mathsf{tm}(B\, a) \mid \mathsf{syn} \hookrightarrow \mathsf{app}\, e\})$$

$$\mathsf{lam} : \{((a : \mathsf{tm}\, A) \to \mathsf{tm}(B\, a)) \to \mathsf{tm}(\mathsf{pi}\, A\, B) \mid \mathsf{syn} \hookrightarrow \mathsf{lam}\}$$

$$\mathsf{lam}\, f = (\mathsf{lam}\, f, f)$$

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- Function extensionality, which also takes place judgmentally.
- Implicit coercion of extension types

Mechanizing this proof in any intensional proof assistant quickly gets out of hand!

"transport hell"

Need a proof assistant with actual equality reflection!

Friendship ended with MUBASIR

TRANSPORT

Now

EQUALITY

REFLECTION

is my

best friend

Istari

https://istarilogic.org

A NuPRL-style [2] proof assistant based on computational semantics developed by Karl Crary.

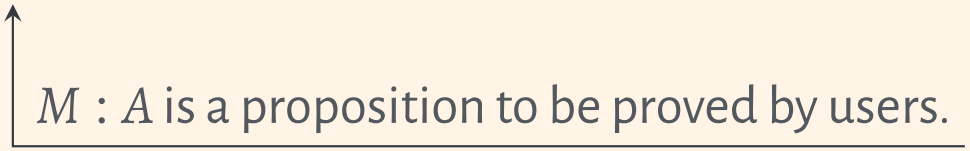
PER (modest set) model over operational semantics, à la logical relations.

Allows: ill-typed terms, terms with multiple types, untyped reduction, etc[1].

[1] A great resource on computational semantics is Angiuli [1].

Istari

- Typing judgment is internal to the language.



Istari

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  $M : A$ is a proposition to be proved by users.

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  Propositional equality! Equality reflection in action.

Istari

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  Propositional equality! Equality reflection in action.

- Type-checking is undecidable!

  If typechecker is stuck, user can mannually take over!

Istari example

Istari

```
define/id_vec {m n}/
/
  fn v . v
//
  forall(m n : nat). vec(m + n) → vec(n + m)
/;
  unfold /id_vec/.
  introOf /m n v/.
  typecheck.
  apply /Nat.plus_commute/.
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a proof that $m + n = n + m$!

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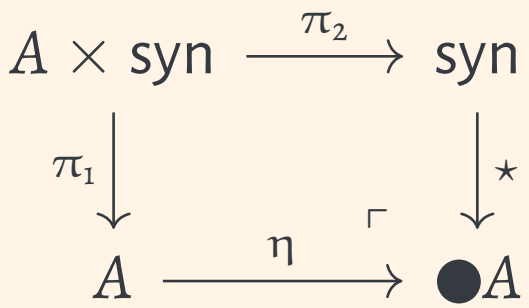
● modality



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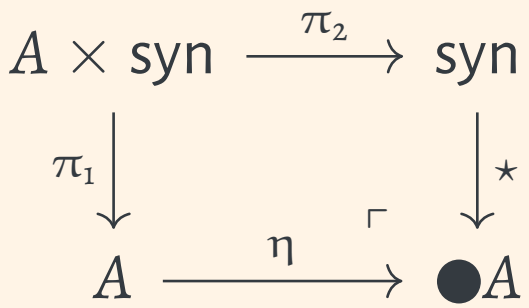

(Non-cubical) Agda

$\bullet : \mathsf{Type} \to \mathsf{Type}$

$\eta : A \to \bullet\, A$

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$\eta{\equiv}\star : (a : A)\ (z : \mathsf{syn}) \to \eta\, a \equiv \star\, z$

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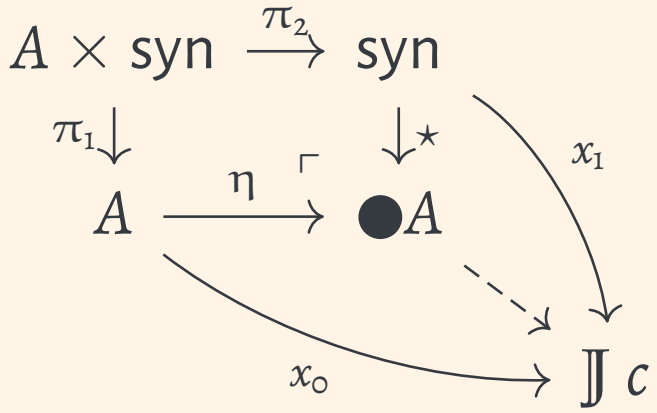

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      (𝕁 : ● A → Type)
      (x0 : (a : A) → 𝕁 (η a))
      (x1 : (z : syn) → 𝕁 (⋆ z))
      ((a : A) → (z : syn) → (subst 𝕁 (η=⋆ a z) (x0 a)) ≡ x1 z) →
      𝕁 c
```
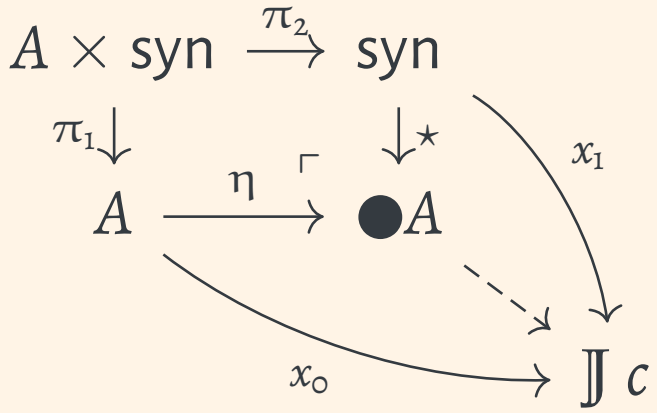

● modality



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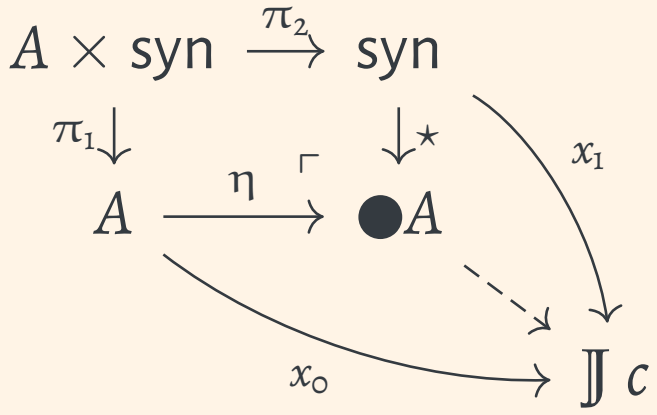

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modality in Istari

Istari

$$
\begin{aligned}
&\mathit{ind} : \mathsf{forall}(c : \mathit{closed}\ A) \\
&\quad (J : \mathit{closed}\ A \to \mathsf{type}) \\
&\quad (x0 : \mathsf{forall}(a : A).\ J(\mathit{eta}\ a)) \\
&\quad (x1 : \mathsf{forall}(z : \mathit{syn}).\ J(\mathit{star}\ z))\ . \\
&\quad (\mathsf{forall}(a : A)(z : \mathit{syn}).\ (x0\ a) = (x1\ z) : \_) \xrightarrow{g} \\
&\quad J\ c
\end{aligned}
$$

modality in Istari

Istari's interative panel

$$
\begin{aligned}
&\mathit{closed} : \mathsf{type} \to \mathsf{type}\\
&\mathit{eta} : A \to \mathit{closed}\ A\\
&\mathit{star} : \mathit{syn} \to \mathit{closed}\ A\\
&\mathit{law} : \mathsf{forall}(z : \mathit{syn})(a : A).\ \mathit{eta}\ a = \mathit{star}\ z : \mathit{closed}\ A\\
&J : \mathit{closed}\ A \to \mathsf{type}\\
&c : \mathit{closed}\ A\\
&x_0 : \mathsf{forall}(a : A).\ J(\mathit{eta}\ a)\\
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&|\!-\\
&\mathit{eta}\ a = \mathit{star}\ z : \mathit{closed}\ A\\
&1\ \mathsf{goal}(\mathsf{depth}\ 0)
\end{aligned}
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\end{aligned}
$$

Use either apply */law/* or auto tactics to fill in the goal.

A library of phase distinction in Istari

- Phase
- Modalities
- Strict glue types
- Extension types

Thanks to Istari's subset type $\{x : A \mid P(x)\}$,
which handles the coercion implicitly: if $a : \{x : A \mid P(x)\}$, then $a : A$.

Mechanizing Synthetic Tait Computability

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Istari

```
define/LAM {A B}/
/
  fn f .glue (fn z .lam f) f
//
Ext ((forall(a : TM A).TM(B a)) → TM(PI A B)) (fn z .lam)
/;
  ...
qed().
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Mechanizing Synthetic Tait Computability

$$\mathsf{lam} : \{((a : \mathsf{tm}\, A) \to \mathsf{tm}(B\, a)) \to \mathsf{tm}(\mathsf{pi}\, A\, B) \mid \mathsf{syn} \hookrightarrow \mathsf{lam}\}$$

$$\mathsf{lam}\, f = (\ \mathsf{lam}\, f\ , f)$$

Istari

```
define/LAM {A B}/
/
  fn f .glue ( fn z .lam f ) f
//
Ext ((forall(a : TM A).TM(B a)) → TM(PI A B)) (fn z .lam)
/;
  ...
qed().
```

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```

Our mechanized proof is almost verbatim the same as the on-paper one!

https://github.com/runmingl/istari-stc

Istari features I didn't talk about

- Quotient types
- Guarded recursive types
- Future modality
- Impredicative quantification
- . . .

Future work

All mechanized in Coq by Karl Crary

Justify phase distinction computationally in Istari's metatheory.

Future work

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Justify phase distinction computationally in Istari's metatheory.

Kripke logical relations over operational semantics

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